Exercise 87

A particle moves along a straight line with displacement $s(t)$, velocity $v(t)$, and acceleration $a(t)$. Show that

$$a(t) = v(t)\frac{dv}{ds}$$

Explain the difference between the meanings of the derivatives dv/dt and dv/ds.

Solution

dv/dt is the rate that velocity changes as time increases, whereas dv/ds is the rate that velocity changes as displacement increases. The acceleration is the derivative of velocity, normally written as

$$a(t) = \frac{dv}{dt}.$$

But if the velocity is known as a function of position rather than time, that is,

$$v = v(s),$$

then

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}[v(s)] = v'(s) \cdot \frac{d}{dt}(s) = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v(t)$$

by the chain rule.